Energy-balance / snow-melt modeling exercise *v 1.0*

The point of this exercise (yes – it really does have a point!) is to gain some appreciation of the different factors involved in snow melt and hence on one of the primary controls on water resources and incident of flooding below glaciated catchments. It is also a good exercise for thinking hard about models (particularly empirical models) and their inherent assumptions, and sources of error and uncertainty.

Your goal is to construct a simple snow melt model for a generic tree-free location on the Arolla valley floor, which you will drive using hourly measurements made at the same location, of: air temperature and windspeed, relative humidity and atmospheric pressure, and estimated cloud cover. You will also measure downwards (incoming, solar) and upwards (reflected) shortwave radiation and hence be able to calculate the albedo of the snow surface (as well as of other surfaces). Finally, you will make direct measurements of snow-pack shrinkage, to test to what extent your model snowmelt calculations and assumptions are realistic (or not). (As a small aside, you will scale your model up to the entire catchment by using estimated areas of snow cover, with and without tree cover, to come up with an overall estimate of the potential contribution of total catchment snowmelt to the stream flow.)

In summary – you will: (i) measure a series of metrological and energy budget variables, (ii) construct a generic energy-balance / snow-melt model in Excel and use this to calculate the snow-melt during the day at your measurement site, (iii) evaluate your model against snow surface height measurements made during the day.

Key papers are provided (also do some reading!) on the website. For instance, *Sicart et al.* [2005] present some measured and derived time-series of some of the various different components of the snow surface energy budget, and these can provide you with an important order-of-magnitude reality check on the energy budget components you calculate in your model. Printouts of some (somewhat random and not necessarily particularly useful) papers are available plus a textbook on environmental physics.

Data (metrological measurements)

You will be making hourly (or more frequent) measurements of:

- Windspeed (m s^{-1})
- Temperature (°C)
- Relative humidity (%)
- Atmospheric pressure (mbar)
- Precipitation (mm (per hour)

using a simple weather station setup.

You also make hourly (or more frequent) measurements of:

- Incoming shortwave (SW) radiation (solar radiation)
- Outgoing SW
- Incoming longwave (LW) radiation
- Outgoing LW

The measurements of incoming and outgoing SW and LW radiation are actually recorded as a voltage (in units of mV). Both sensors have been calibrated to convert mV to shortwave radiation.

Their sensitivities (which are also written on the underside of the sensor, although in units of μV (10^{-6} V) per (W m^{-2})) are:

upper sensor (incoming SW): 0.01463 mV per (W m^{-2})
upper sensor (outgoing SW): 0.01443 mV per (W m^{-2})

upper sensor (incoming LW): 0.00795 mV per (W m^{-2})
upper sensor (outgoing LW): 0.01312 mV per (W m^{-2})

In other words – to get the energy flux (W m^{-2}) you simply divide your measured voltage (in mV) by the respective sensor's sensitivity. As a reality check on the incoming shortwave solar radiation – the calibration for the instrument was carried out in a similar Swiss mountain location in the summer and reported a range of solar radiation of between 645 and 1043 W m^{-2} under mostly clear-sky conditions. (You can check your incoming and outgoing LW measurements using the equations in the supplemental handout.)

At the same site(s) at which you make the metrological measurements, you will measure the evolution of the height of the snow-pack surface simply by placing an upright ruler in the snow and recording how the height of the snow evolves during the day. Strictly, you only need to know the height at the start and end of the day, but in case the ruler falls over or is attacked by a marmot, it is safer to record hourly alongside all the other measurements. If the rate of change in snow height and hence melt rate changes during the course of the day, this might be quite interesting to discuss the reasons for. In case there is preferential melt in the vicinity of the ruler, you may need to use a flat piece of paper to extrapolate the snow-pack height to the ruler scale.

Snow-melt modelling

(a) Energy-balance / snow-melt model

Ignoring that we might have to heat up the snow to melting point (zero degrees C) first (how important might this be, and particularly: are there specific times of the day when it might significantly change things?), the energy available for melt is given by the energy inputs minus the energy loses. Energy gains include:

- Incoming solar radiation (shortwave – SWR_{in})
- Absorbed longwave (LWR_{in})
- Turbulent sensible heat transfer from the air (SHF), and heat energy from rain

Loses include:

- Outgoing (reflected) shortwave radiation (SWR_{out})
- Emitted longwave (LWR_{out})
- Turbulent moisture loss from the surface (LHF) (and heat lost in sublimating the ice and transferring the moisture to the atmosphere). (Note that ice sublimation is regarded as a loss in that it represents energy used that is then not available to drive melting and melt-water generation.)

A basic energy budget is given in *Konzelmann and Braithwaite* [1995] – Equation (10), although in this case it is written in terms of the energy for 'ablation' of a glacier. *Konzelmann and Braithwaite* [1995] also take into account heat conduction which we will not worry about here (we are effectively assuming that the snow-pack and underlying ground are always at 0°C) and include an error term, which we will also not worry about. The notation for some of the fluxes may also be different.

In summary: Your energy budget (heat available for melting snow) consists of 6 terms. 4 you measure directly (SWR_{in} and SWR_{out}, LWR_{in} and LWR_{out}), and 2 you calculate using equations in *Konzelmann and Braithwaite* [1995] (SHF and LHF).

In more detail:

1. Following the notation of *Konzelmann and Braithwaite* [1995] – SWR is your absorbed shortwave input. In the paper it is described in terms of net shortwave, which is incoming minus outgoing, and equivalent to incoming times ($1 - \alpha$) where α is the snow albedo. However, you have actual measurements of incoming and outgoing and so can simply difference the two instead if you prefer, i.e. SWR_{in} - SWR_{out}.
2. The terms, SHF and LHF are calculated following *Konzelmann and Braithwaite* [1995] (Equations 3-6). All met variables and constants are given, with the exception of vapour pressure and saturated vapour pressure (e_s) – the estimation of the latter is given above, while the former is simply relative humidity (a met variable) as a fraction, multiplied by e_s.
   • Watch out for units here – the equation for saturated water vapour pressure (e_s) given in the supplementary handout gives a number in units of kPa, whereas Equation 5 in *Konzelmann and Braithwaite* [1995] is expecting units of Pa (as per the units for standard atmospheric pressure).
   • The '*difference between vapour pressure at 2 m … and at the [snow] surface*', Δe_2, is given by:
   $\Delta e_2 = e_s - e_a$
   where e_s is the saturated vapour pressure at the snow surface (and at a temperature of 0°C). The vapour pressure at 2 m (e_a) is equal to the relative humidity (as a fraction) that you have measured, multiplied by the saturated vapour pressure (e_s) at your measured air temperature, i.e.
   $e_a = (RH/100) \times e_s$
   (Relative humidity is actually defined as the proportion of water vapour in the air compared to that when saturated.)
   • For SHF (Equation 3) – the air temperature variable in the equation (T_2) i in effect the difference between the air temperature and 0°C (or simply the observed temperature in units of °C). Note that you will only get net turbulent heat transfer to the snow surface when the air temperature is above 0°C, otherwise the air will cool the snow surface.
3. The term not included in *Konzelmann and Braithwaite* [1995] – energy input from rain, can be calculated as the difference between rain temperature (assume the same as air temperature) and the melting point of ice (0°C) multiplied by the specific heat capacity. You will need to convert mm (per hour) into kg m^{-2} of water flux. (Use the density of water to get kg and calculate the volume of water per m^2 corresponding to the specific number of mm of rainfall.) You are unlikely to be getting any rain, however …
4. When/if checking the values of longwave radiation – the two components of longwave (LWR) are estimated as per the *supplementary handout on longwave calculations*. Note that the value of the Stefan–Boltzmann constant (σ) in SI units is 5.670373×10^{-8} W m^{-2} K^{-4}.

In Excel – in calculating the various energy budget components each hour, you should aim for something looking a little like Table 2 of *Konzelmann and Braithwaite* [1995], except at an hourly resolution rather than daily. **The key is to work through methodically**. Identify the most important factors first and calculate those. Break down complex equations into components which are calculated in separate Excel columns. You can make a start with just the initial measurements.

(b) Testing your model

You have made equivalent measurements of metrological properties, and at hourly intervals, to the historical observations provided. Critically, you have also measured snow melt. You can then use the same equations as in **(a)** to calculate the hourly energy budget and hence total snowmelt.

Calculate the actual snowmelt from the measured loss in snow surface height, converted to a mass of water per m^2, taking into account the porosity (or density) of the snow. Porosity (or density) can be measured by weighing a known volume of snow. For calculating the energy available for snowmelt: units of W m^{-2} are equivalent to J s^{-1} m^{-2}. Each hour you have 3600 s. The energy required to melt 1 kg of ice (the latent heat of fusion of ice) is approximately: 334000 J kg^{-1}. You can hence calculate the mass (kg) of snow melt per hour (and total over the day).

(c) Extrapolating your model

For (optional) added fun, you might extrapolate your snowmelt model to the entire catchment. To do this you will need to estimate the area of bare snow, and also the area of tree-covered snow, from your sketch of the catchment (from a line above the hotel) and a map (on-line or printed) to provide the scale. Start by scaling up your calculated melt per m^2 (your calculations made as part of **(a)** and **(b)**) to the estimated area of bare snow in the valley. Discard the area of bare rock. Be intelligent with what you do about wooded areas. You may have measured the incoming SWR in some typical wooded areas and compared to bare ground to estimate the radiation interception by the tree canopy (which is then not available for snow melt). You may have to make some crude assumptions about wind speed in wooded areas compared to your measurements at an open site (and similarly for air temperature).